
Construction of a generalized psychrometric chart for different pressures

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Abstract In textbooks of thermodynamics or air conditioning engineering we can find the psychrometric chart at atmospheric pressure. However, standard psychrometric charts cannot be used directly in those areas where urban elevations and barometric pressures are considerably different from those given on the charts. Based on the formulas suggested by ASHRAE, a generalized psychrometric chart for different pressures is constructed, using composite thermodynamic properties ϕ/p , p_w/p , and vp . A nomograph is given to relate these composite thermodynamic properties to the psychrometric variables ϕ , p_w and v with p . Other commonly used psychrometric variables can be read directly from the generalized psychrometric chart. Three numerical examples illustrate the use of the generalized psychrometric chart. It will be of material help in both the design of air conditioning systems for different barometric pressures and the teaching of psychrometrics.

Keywords moist air; psychrometric chart; barometric pressure

Nomenclature

h specific enthalpy of moist air
 h_a specific enthalpy of dry air
 h_w specific enthalpy of water vapor
 m_a mass of dry air
 m_w mass of water vapor
 n number of moles
 n_a number of moles of dry air
 n_w number of moles of water vapor
 p total pressure of moist air
 p_a partial pressure of dry air
 p_w partial pressure of water vapor
 p_{ws} saturation pressure of water vapor
 p_w/p composite partial pressure of water vapor
 R universal gas constant
 R_a gas constant for dry air
 R_w gas constant for water vapor
 t temperature, dry-bulb temperature
 t_d dewpoint temperature
 t_{wb} thermodynamic wet-bulb temperature
 T absolute temperature
 v specific volume of moist air
 vp composite specific volume

V volume
 w humidity ratio
 x_a mole fraction of dry air
 x_w mole fraction of water vapor
 ε enthalpy–moisture ratio
 ρ_w density of water vapor
 ϕ relative humidity
 ϕ/p composite relative humidity

Introduction

The thermodynamic properties of moist air are basic to some technologies, such as heating and air conditioning, drying, food storage and meteorology. A psychrometric chart graphically represents the thermodynamic properties of moist air. Such a chart not only allows graphical reading of the various properties but also provides convenient graphical solutions to many problems involving moist air processes. Traditionally, a psychrometric chart may be constructed for some single value of barometric pressure, usually standard sea-level pressure. ASHRAE have developed seven such psychrometric charts for sea-level pressure (101.325 kPa), 750-m altitude (92.66 kPa), 1500-m altitude (84.54 kPa), and 2250-m altitude (77.04 kPa) [1]. However, these standard charts cannot be used directly in those areas where urban elevations and barometric pressures are considerably different from those given on the charts [2]. It is also difficult to use these charts for moist air processes during which the pressure varies. It is necessary to construct a generalized psychrometric chart for various pressures.

For a given barometric pressure, all psychrometric properties of moist air are determined by two independent variables. Usually two basic coordinates of the psychrometric chart are enthalpy and humidity ratio. Other lines of constant properties, such as dry-bulb and wet-bulb temperatures, relative humidity, specific volume, partial pressure of water vapor, and so on can then be plotted on the chart as a function of enthalpy and humidity ratio. In the present paper, a generalized psychrometric chart covering all possible pressure cases in air conditioning design is constructed.

Thermodynamic properties of moist air

When moist air is considered a binary mixture of independent perfect gases, namely dry air and water vapor, each component and the mixture also are assumed to obey the perfect gas equation of state, as follows:

$$\text{Dry air} \quad p_a V = n_a RT = m_a R_a T \quad (1)$$

$$\text{Water vapor} \quad p_w V = n_w RT = m_w R_w T \quad (2)$$

$$\text{Moist air} \quad pV = nRT \quad (3)$$

where $p = p_a + p_w$ is the total mixture pressure and $n = n_a + n_w$ is the total number of moles in the mixture. The mole fraction of dry air and water vapor can be expressed as the ratio of the partial pressure to the total pressure, respectively:

$$x_a = n_a/n = p_a/p \quad (4)$$

$$x_w = n_w/n = p_w/p \quad (5)$$

The humidity ratio of a given moist air sample is defined as the ratio of the mass of water vapor to the mass of dry air contained in the sample. From equations (1) to (5), the humidity ratio, w , is given by:

$$w = \frac{m_w}{m_a} = 621.98 \frac{p_w}{p - p_w} = 621.98 \frac{x_w}{1 - x_w} \quad (\text{g/kg}_a) \quad (6)$$

Absolute humidity (alternatively, water vapor density) is the ratio of the mass of water vapor to the total volume of the sample:

$$\rho_w = \frac{m_w}{V} \quad (\text{kg/m}^3) \quad (7)$$

Relative humidity is the ratio of the moist air's absolute humidity to the absolute humidity of saturated air at the same temperature. From equations (2) and (5), relative humidity can be written:

$$\phi = \frac{\rho_w}{\rho_{ws}} = \frac{x_w}{x_{ws}} = \frac{p_w}{p_{ws}} \quad (8)$$

where the term p_{ws} represents the saturation pressure of water vapor at the given temperature. This pressure is a function only of temperature and can be calculated from following formulas, published by Hyland and Wexler [3]:

$$\ln(p_{ws}) = \sum_{i=0}^5 c_i T^{i-1} + c_6 \ln(T) \quad (9)$$

where the coefficients for the temperature range $173.16 \leq T \leq 273.15 \text{ K}$ are

$$\begin{aligned} c_0 &= -5.6745359 \times 10^3, & c_1 &= 6.3925247, & c_2 &= -9.6778430 \times 10^{-3}, \\ c_3 &= 6.2215701 \times 10^{-7}, & c_4 &= 2.0747825 \times 10^{-9}, & c_5 &= -9.4840240 \times 10^{-13}, \\ c_6 &= 4.1630159 \end{aligned}$$

and for $273.16 \leq T \leq 473.15 \text{ K}$ are:

$$\begin{aligned} c_0 &= -5.8002206 \times 10^3, & c_1 &= 1.3914993, & c_2 &= -4.8640239 \times 10^{-2}, \\ c_3 &= 4.1764768 \times 10^{-5}, & c_4 &= -1.4452093 \times 10^{-8}, & c_5 &= 0, & c_6 &= 6.5459673 \end{aligned}$$

The specific volume of a moist air mixture is expressed in terms of a unit mass of dry air. Using equations (1) and (6):

$$v = \frac{V}{m_a} = \frac{R_a T}{p - p_w} = \frac{R_a T}{p} \left(1 + \frac{w}{621.98} \right) \quad (\text{m}^3/\text{kg}_a) \quad (10)$$

The enthalpy of moist air equals the sum of the enthalpies of the dry air and the water vapor. The specific enthalpy of moist air is also defined per unit mass of dry

air. For lower pressure, the specific enthalpy of water vapor is approximately the linear function of temperature. Therefore, the enthalpy of moist air can be expressed as [4]:

$$h = h_a + wh_w = 1.005t + w(2.501 + 1.805 \times 10^{-3}t) \quad (\text{kJ/kg}_a) \quad (11)$$

where t is the dry-bulb temperature, in °C.

Another commonly used psychrometric parameter is the wet-bulb temperature. In principle, there is a difference between the wet-bulb temperature and the thermodynamic wet-bulb temperature (otherwise known as the temperature of adiabatic saturation). The wet-bulb temperature measured by a wet-bulb thermometer is a function of both heat and mass diffusion rates, while the thermodynamic wet-bulb temperature is a function of a thermodynamic equilibrium process. During the process, water by evaporating into air can bring the moist air to adiabatic saturation at the thermodynamic wet-bulb temperature. The energy equation for this process is

$$h + (w_s - w)h_{wb} = h_s \quad (12)$$

where the specific enthalpy and humidity ratio of moist air are respectively increased from given initial values h and w to the values h_s and w_s , corresponding to saturation at the thermodynamic wet-bulb temperature, t_{wb} . The specific enthalpy, h_{wb} , of the water added at temperature t_{wb} can be calculated by the approximate relation

$$h_{wb} = 4.186t_{wb} \quad (\text{kJ/kg}) \quad (13)$$

Construction of the generalized psychrometric chart

A thermodynamic state for moist air is uniquely fixed if the barometric pressure and two independent properties mentioned above are known. Traditionally, a psychrometric chart is constructed for some single value of barometric pressure, usually standard sea-level pressure. In fact, a generalized psychrometric chart for different pressures can be constructed as long as the psychrometric parameters such as ϕ , p_w , and v are substituted by composite parameters involved with the total pressure, respectively known as the composite relative humidity, ϕ/p , composite partial pressure of water vapor, p_w/p , and composite specific volume, vp .

Enthalpy lines (h) and humidity ratio lines (w)

As in the ASHRAE charts, the enthalpy and humidity ratio are used as the basic coordinates in the generalized psychrometric chart for different pressures, as shown in Fig. 1. The other properties or composite parameters can then be represented by the enthalpy and humidity ratio with the aid of the equations given above. Humidity ratio lines are horizontal, and enthalpy lines are obliquely drawn across the chart so that other property lines are more distinct and can be read more accurately.

Dry-bulb temperature lines (t)

According to equation (11), enthalpy is the linear function of the humidity ratio for constant dry-bulb temperature. Thus, the dry-bulb temperature lines are straight but

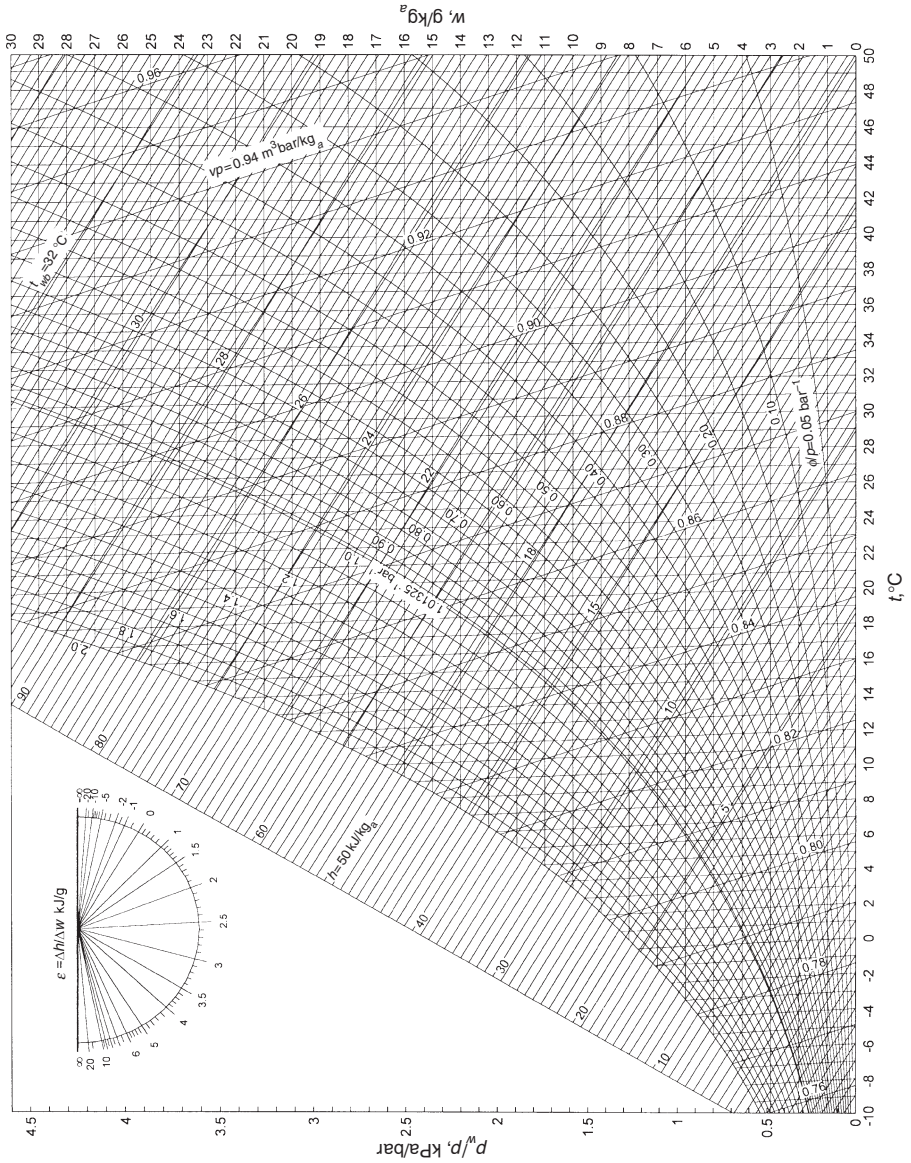


Fig. 1 Psychrometric chart for different pressures.

are not parallel to each other, since their slopes vary with temperature. From equation (11) we have $h = 1.005t$ for $w = 0$, that is, the values of the enthalpy and dry-bulb temperature on the abscissa ($w = 0$) are approximately equal. These properties are also true for different pressures because there is no pressure term in equation (11).

Composite partial pressure of water vapor (p_w/p)

From equations (5) and (6), we see that the composite partial pressure of water vapor is synonymous with the mole fraction of water vapor, and w is the monodromic function of p_w/p . Thus, constant lines of composite partial pressure of water vapor are consistent with constant humidity ratio lines, from equation (6).

Composite relative humidity lines (Φ/p)

From equations (6) and (8), we obtain:

$$\frac{\phi}{p} p_{ws}(t) = \frac{w}{621.98 + w} \quad (14)$$

where the saturation pressure of water vapor, $p_{ws}(t)$, is a function only of temperature. Substituting equation (9) for p_{ws} and combining with equation (11) to eliminate t , the relationship between the enthalpy and the humidity ratio for constant composite relative humidity, Φ/p , can be found by:

$$\ln\left(\frac{\phi}{p}\left(1 + \frac{621.98}{w}\right)\right) + \sum_{i=0}^5 c_i \left(\frac{h - 2.501w}{1.005 + 1.81 \times 10^{-3}w} + 273.15\right)^{i-1} + c_6 \ln\left(\frac{h - 2.501w}{1.005 + 1.81 \times 10^{-3}w} + 273.15\right) = 0 \quad (15)$$

Then, lines of constant composite relative humidity can be drawn on $h-w$ coordinates. Every line for ϕ/p may be either a saturation curve ($\phi = 100\%$) or an unsaturation curve ($\phi < 100\%$), which depends on the pressure. For example, the line for $\phi/p = 1 \text{ bar}^{-1}$ is a saturation curve for pressure $p = 1 \text{ bar}$, but an unsaturation curve ($\phi = 50\%$) for $p = 0.5 \text{ bar}$. For $p = 0.5 \text{ bar}$ the line of $\phi/p = 2 \text{ bar}^{-1}$ is its saturation curve. The saturation curve at standard atmospheric pressure can be represented by the constant line of $\phi/p = 1.01325^{-1} \text{ bar}^{-1}$ as shown in Fig. 1.

Composite specific volume lines (vp)

The relationship between the enthalpy and the humidity ratio for a constant composite specific volume can be obtained by substituting equation (10) for t in equation (11):

$$h = 2.501w + \frac{vp}{R_a} \left(\frac{1.005 + 1.81 \times 10^{-3}w}{1 + 1.6078 \times 10^{-3}w}\right) - 273.15(1.005 + 1.81 \times 10^{-3}w) \quad (16)$$

It is shown from equation (16) that lines of vp are nearly straight, because, in the second term of the right side of the equation, the values of the numerator and denominator in parenthesis are approximately equal.

Thermodynamic wet-bulb temperature lines (t_{wb})

Substituting equations (11), (6), and (13) respectively for h_s , w_s , and h_{wb} , equation (12) becomes:

$$h = 4.186t_{wb} \cdot w + 1.005t_{wb} + 621.98 \frac{\frac{\phi}{p} p_{ws}(t_{wb})}{1 - \frac{\phi}{p} p_{ws}(t_{wb})} (2.501 - 4.18419t_{wb}) \quad (17)$$

where the saturation pressure, $p_{ws}(t_{wb})$, of water vapor at t_{wb} can be calculated from equation (9). It will be seen from equation (17) that lines of constant thermodynamic wet-bulb temperature, t_{wb} , are straight but are not parallel to each other, since the slope varies with t_{wb} . On the other hand, the slope of t_{wb} lines do not depend on the pressure, p , and therefore thermodynamic wet-bulb temperature lines for a given t_{wb} may be parallel to each other regardless of different ϕ/p , because they have the same slope. Thus, we can arbitrarily choose a value of ϕ/p to plot thermodynamic wet-bulb temperature lines by equation (17). For instance, selecting $\phi/p = 1 \text{ bar}^{-1}$ (the saturation line for $p = 1 \text{ bar}$), the lines of thermodynamic wet-bulb temperature corresponding to $p = 1 \text{ bar}$ are shown in Fig. 1. After determining the saturation curve for other given pressures, thermodynamic wet-bulb temperature lines corresponding to the pressure are drawn through intersections of constant dry-bulb temperature lines $t = t_{wb}$ and the saturation curve, and parallel to t_{wb} lines of $p = 1 \text{ bar}$.

Enthalpy–moisture ratio protractor (ε)

The protractor to the left of Fig. 1 shows the enthalpy–moisture ratio, ε , which is defined as the ratio of the total heat to moisture. For a given process from state 1 to state 2, ε may be directly calculated by the ratio of enthalpy difference to humidity ratio difference:

$$\varepsilon = \frac{Q}{W} = \frac{\Delta h}{\Delta w} \quad (18)$$

It follows that the enthalpy–moisture ratio represents the slope of the moist air process line. The protractor may be used to establish the direction of a process line on the psychrometric chart.

The generalized psychrometric chart for different pressure as shown in Fig. 1 can be used to solve heating and air conditioning problems. Ranges of thermodynamic properties covered by the chart are:

$$t = -10\text{--}50^\circ\text{C}, \quad w = 0\text{--}30 \text{ g/kg}_a, \quad h = 0\text{--}130 \text{ kJ/kg}_a, \quad t_{wb} = 0\text{--}34^\circ\text{C},$$

$$p_w/p = 0\text{--}4.6 \text{ kPa/bar}, \quad \phi/p = 0.05\text{--}2.0 \text{ bar}^{-1}, \quad v/p = 0.76\text{--}0.96 \text{ m}^3 \text{ bar/kg}_a.$$

Fig. 2 is a nomograph, in which composite parameters p_w/p , ϕ/p , and v/p are related to p_w , ϕ , and v with p . After obtaining composite parameters from Fig. 1, values of p_w , ϕ , and v can be determined either by calculation or by reading directly from Fig. 2.

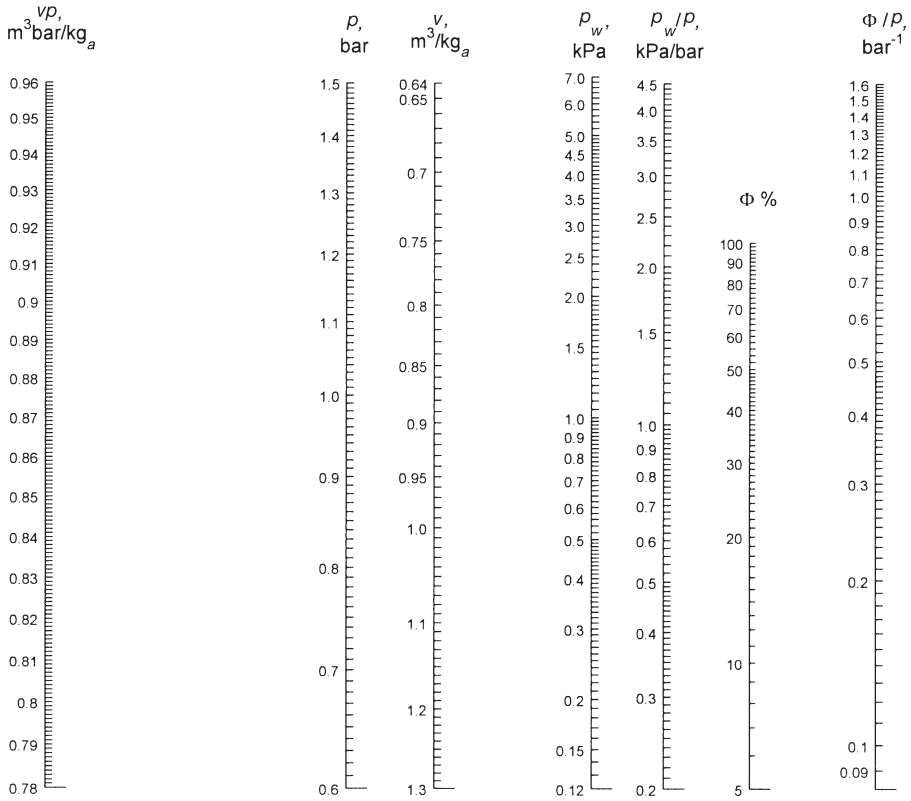


Fig. 2 A nomograph of composite parameters.

Numerical examples

Example 1

Moist air exits at dry-bulb temperature $t = 25\text{ }^\circ\text{C}$, relative humidity $\phi = 50\%$. Determine the other moist air properties for pressures 101.325, 77.058, and 10.5263 kPa.

Solution

Composite relative humidity ϕ/p corresponding to the saturation state ($\phi = 100\%$) can be read from Fig. 2 for three known pressures: $1.01325^{-1}\text{ bar}^{-1}$, 1.3 bar^{-1} , and 0.95 bar^{-1} . The saturation curve corresponding to each pressure can be found from Fig. 1. Read ϕ/p corresponding to p and $\phi = 50\%$ from Fig. 2.

Locate the state point on Fig. 1 at the intersection of the $25\text{ }^\circ\text{C}$ dry-bulb temperature line and the ϕ/p line. Read values of h , w , p_w/p , and vp , directly. Then the partial pressure of the water vapor, p_w and specific volume, v , can be read from Fig. 2. The thermodynamic wet-bulb temperature, t_{wb} , can be read at the intersection of the saturation curve and the line of constant thermodynamic wet-bulb temperature through the state point. The dew point temperature, t_d , can be found at the intersection of the

TABLE 1 Solutions to numerical example 1

p , Pa	101 325	77 058	105 263
t , °C	25	25	25
ϕ	50%	50%	50%
ϕ/p , bar ⁻¹	0.4935	0.6489	0.4750
p_w/p , kPa/bar	1.57	2.06	1.51
νp , m ³ bar/kg	0.8695	0.8735	0.8690
h , kJ/kg _a	50.3	58.4	49.4
w , g/kg	9.9	13.1	9.5
p_w , kPa	1.59	1.59	1.59
ν , m ³ /kg	0.858	1.134	0.8256
t_{wb} , °C	17.8	17.2	18
t_d , °C	13.9	13.9	13.9

line of constant w and the saturation curve. Table 1 lists all the moist air properties for the three pressures.

Example 2

Given dry-bulb temperature $t = 40^\circ\text{C}$ and thermodynamic wet-bulb temperature $t_{wb} = 20^\circ\text{C}$, determine the other properties of moist air for pressures of 101.325, 77.058, and 105.263 kPa.

Solution

As in example 1, the ϕ/p values of saturation state for the three pressures are 1.01325^{-1} bar⁻¹, 1.3 bar⁻¹, 0.95 bar⁻¹, respectively. Then, the saturation curves corresponding to the above three ϕ/p values can be found from Fig. 1. Parallel to the $t_{wb} = 20^\circ\text{C}$ line of $p = 1$ bar, the thermodynamic wet-bulb temperature line corresponding to the above pressures is drawn through the intersection of the $t = 20^\circ\text{C}$ dry-bulb temperature line and the saturation curve. The intersection of this thermodynamic wet-bulb temperature line with the 40°C dry-bulb temperature line is the state point. Read h , w , and composite parameters ϕ/p , p_w/p , νp directly. Then, ϕ , p_w , and ν can be found from Fig. 2. All the properties of the moist air are listed in Table 2.

Example 3

A stream at $p_1 = 125$ kPa, $t_1 = 13^\circ\text{C}$ and $\phi_1 = 62.5\%$ is adiabatically mixed with the other stream at $p_2 = 111$ kPa, $t_2 = 32^\circ\text{C}$ and $\phi_2 = 50\%$. The two mass flows of dry air are the same, $m_{a1} = m_{a2}$. Find the dry-bulb temperature t_3 , the thermodynamic wet-bulb temperature t_{wb3} , and the relative humidity, ϕ_3 , of the resulting mixture when the pressure is decreased to $p_3 = 101.325$ kPa.

Solution

The composite relative humidities of the two streams in the initial state can be determined by:

TABLE 2 Solutions to numerical example 2

p , Pa	101325	77058	105263
t , °C	40	40	40
t_{wb} , °C	20	20	20
ϕ/p , bar ⁻¹	0.141	0.2375	0.1270
p_w/p , kPa/bar	1.03	1.75	0.93
νp , m ³ bar/kg	0.908	0.9145	0.9073
h , kJ/kg _a	56.8	68.9	55.3
w , g/kg	6.5	11.2	5.86
ϕ	14.3%	18.3%	13.4%
p_w , kPa	1.044	1.349	0.983
ν , m ³ /kg	0.9861	1.1868	0.8619
t_{ds} , °C	7.6	11.5	6.5

$$\frac{\phi_1}{p_1} = \frac{0.625}{1.25 \times 10^5} = 0.5 \text{ bar}^{-1}, \quad \frac{\phi_2}{p_2} = \frac{0.50}{1.11 \times 10^5} = 0.45 \text{ bar}^{-1}$$

State 1 and state 2 are located at the intersections of t_1 , and ϕ_1/p_1 , t_2 , and ϕ_2/p_2 , respectively. Thus, $h_1 = 25 \text{ kJ/kg}_a$, $w_1 = 4.7 \text{ g/kg}_a$, $h_2 = 67 \text{ kJ/kg}_a$, $w_2 = 13.6 \text{ kJ/kg}_a$. The adiabatic mixing process is governed by the energy and mass balance equations. Therefore:

$$\frac{h_2 - h_3}{h_3 - h_1} = \frac{w_2 - w_3}{w_3 - w_1} = \frac{m_{a,1}}{m_{a,2}}$$

Note that $m_{a,1} = m_{a,2}$. The specific enthalpy and the humidity ratio of the final state can be calculated from the above equation: $h_3 = 46 \text{ kJ/kg}_a$, $w_3 = 9.15 \text{ g/kg}_a$. State 3 is located on Fig. 1, and the values $t_3 = 22.6^\circ\text{C}$ and $\phi_3/p_3 = 0.53 \text{ bar}^{-1}$ can be read off the chart. Thus $\phi_3 = 0.53 \times 1.01325 = 53.7\%$. Thermodynamic wet-bulb temperature, t_{wb3} , is found by drawing the line of the thermodynamic wet-bulb temperature through state 3, and intersecting with the saturation curve for $\phi/p = 1.01325^{-1} \text{ bar}^{-1}$, to give $t_{wb3} = 16.3^\circ\text{C}$.

Conclusions

Using composite thermodynamic properties ϕ/p , p_w/p , and νp , a generalized psychrometric chart for different pressures is constructed. Compared with the standard psychrometric charts for some single value of barometric pressure, the generalized psychrometric chart covers all the possible pressure cases in air conditioning design. The method can be applied to heating and food storage, and can be extended to drying and meteorology.

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